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VIBRATIONAL NONEQUILIBRIUM RADIATION OF DIATOMIC GASES—II

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Abstract—The second part deals with the approximations of the master equation for a vibrational energy distribution of a diatomic gas. The equations of the Biberman–Holstein type are obtained, in which nonequilibrium is approximately described. The diffusion equation is derived and the corresponding boundary condition is formulated. The diffusion coefficient is calculated, which is related to the mean absorption coefficient, differing essentially from the appropriate mean values, introduced according to Planck and Rosseland.

1. DESCRIPTION OF VIBRATIONAL NONEQUILIBRIUM BASED ON INTEGRAL EQUATIONS

The master integro-differential equation (1.46) can also be extended to a multicomponent gas. Thus, for example, for the most interesting case of hot particles radiating in a continuous spectrum, the master equation becomes

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla+\frac{1}{\tau}\right)\varepsilon(\mathbf{r}) = \frac{q}{\rho(\mathbf{r})}\int_{\Delta\nu}\mathrm{d}\nu\int_V \nu^3 k_\nu(\mathbf{r}')k_\nu(\mathbf{r}) \times\frac{\exp(-\tau_\nu(\mathbf{r},\mathbf{r}'))}{|\mathbf{r}-\mathbf{r}'|^2}\varepsilon(\mathbf{r}')\,\mathrm{d}\mathbf{r}' + \frac{1}{\rho(\mathbf{r})}\int_{\Delta\nu}\mathrm{d}\nu\int_V k_\nu^b(\mathbf{r}')k_\nu(\mathbf{r})\frac{\exp[-\tau_\nu(\mathbf{r},\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|^2} \times B_\nu(\mathbf{r}')\,\mathrm{d}\mathbf{r}'+\frac{\varepsilon^0(\mathbf{r},t)}{\tau^0}+\psi(\mathbf{r},t), \quad (1)$$

$$\tau_\nu(\mathbf{r},\mathbf{r}') = \int_0^{|\mathbf{r}-\mathbf{r}'|}(k_\nu+k_\nu^b)\,\mathrm{d}s,\quad \frac{1}{\tau}=\frac{1}{\tau^0}+\frac{1}{\tau^*}, \quad (2)$$

in which

$$k_\nu = h\nu NB\sum_{j=0}^{\infty}\sum_{\alpha=0}^{1}S_{j\alpha}\phi_{j\alpha}(\nu).$$

The equation of radiative transfer in this case is

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\mathbf{n}\nabla\right)J(\mathbf{k}) = k_\nu q\nu^3\varepsilon+k_\nu^b B_\nu-(k_\nu+k_\nu^b)J(\mathbf{k}). \quad (3)$$

The second integral term in (1) is obtained by changing $\hat{J}_\nu$ in the source (I. 48) to the intensity of radiating particles. The vibrational relaxation time is changed as well due to the presence of particles of different type.

Equation (1) is certainly much simpler than the original set (I. 1–3), but further simplification would be of practical interest. Our approach rests on successive approximations of the kernel of equation (1). To avoid tedious mathematics, we shall consider the approximations on the example of a uniform flat layer, while the generalization for the three-dimensional non-uniform case is included in the Appendix.

Let $\mathbf{v}=0$, $k_\nu^b=0$, $\rho=\text{const}$, $T=\text{const}$, then for the flat layer, on integration over the angles ϕ and θ in cylindrical coordinates equation (1) becomes

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\varepsilon(\xi) = \int_0^1 K(|\xi-\zeta|)\varepsilon(\zeta)\,\mathrm{d}\zeta+\Phi(\xi,t), \quad (4)$$

where

$$K(\xi) = \frac{2\pi\delta q}{\rho}\int_{\Delta\nu}\nu^3 k_\nu^2 E_1[\delta k_\nu\xi]\,\mathrm{d}\nu, \quad (5)$$

$$E_1(\xi) = \int_0^1\exp\left(-\frac{\xi}{\mu}\right)\frac{\mathrm{d}\mu}{\mu},\quad \Phi=\frac{\varepsilon^0}{\tau^0}+\psi. \quad (6)$$

Since at low pressures the line-shape functions can be considered as non-overlapping, then with the help of (I. 24–27) one obtains

$$K(\xi) = \frac{2\pi\delta q}{\rho}(hNB)^2\sum_{j\alpha j'\alpha'}S_{j\alpha}S_{j'\alpha'} \times\int_{\Delta\nu}\mathrm{d}\nu\nu^5\phi_{j\alpha}(\nu)\phi_{j'\alpha'}(\nu)E_1[\delta k_\nu\xi] = \frac{2\pi\delta q}{\rho}(hNB)^2\sum_{j\alpha}S_{j\alpha}^2\nu_{j\alpha}^5 \times\int_{\Delta\nu}\mathrm{d}\nu\phi_{j\alpha}^2(\nu)E_1[\delta h\nu_{j\alpha}NBS_{j\alpha}\phi_{j\alpha}(\nu)\xi]. \quad (7)$$

After introducing the new notations

$$\tau_{j\alpha}=\frac{\delta hNB}{\sqrt{\pi}\,\mathrm{d}}S_{j\alpha},\quad \mathrm{d}=\left(\frac{2k_B T}{mc^2}\right)^{1/2},$$

$$\alpha(\omega)=\exp(-\omega^2), \quad (8)$$

$$G(\xi)=\pi^{-1/2}\int_{-\infty}^{+\infty}\alpha^2(\omega)E_1[\xi\alpha(\omega)]\,\mathrm{d}\omega, \quad (9)$$

the kernel $K(\xi)$ assumes the form

$$K(\xi)=\frac{2\pi^{3/2}qd}{\delta\rho}\sum_{j\alpha}\tau_{j\alpha}^2\nu_{j\alpha}^4 G(\xi\tau_{j\alpha}). \quad (10)$$

Write down (9) in the form given in [1]

$$G(\xi) = \int_0^\infty \exp(-\xi/x)U(x)\,\mathrm{d}x, \qquad (11)$$

in which $U(x)$ is a certain function, then

$$K(\xi) = \frac{2\pi^{3/2} q d v_0^4}{\delta\rho} \int_0^\infty W(\xi/x)U(x)\,\mathrm{d}x, \qquad (12)$$

where

$$W(z) = \sum_{j\alpha} \tau_{j\alpha}^2 (\nu_{j\alpha}/\nu_0)^4 \exp(-z\tau_{j\alpha}). \qquad (13)$$

Differentiation of (13) and effective subtraction from the sum of the mean optical thickness of the layer σ, yields approximated equation for $W(z)$

$$\frac{\mathrm{d}}{\mathrm{d}z} W(z) = -\sum_{j\alpha} \tau_{j\alpha}^3 (\nu_{j\alpha}/\nu_0)^4 \exp(-z\tau_{j\alpha}) \simeq -\sigma W(z). \qquad (14)$$

To determine σ and the constant of integration of equation (14), we impose on the function $\tilde{W}(z)$, which is the solution of an approximate equation (14), two conditions

$$W(0) = \tilde{W}(0), \quad \int_0^\infty W(z)\,\mathrm{d}z = \int_0^\infty \tilde{W}(z)\,\mathrm{d}z \qquad (15)$$

Thus, with the help of (14) and (15) one obtains the following definition of the mean optical thickness of the layer in the centres of the lines

$$\sigma = \sum_{j\alpha} \tau_{j\alpha}^2 \nu_{j\alpha}^4 \Big/ \sum_{j\alpha} \tau_{j\alpha} \nu_{j\alpha}^4. \qquad (16)$$

Replacing of $W(z)$ in (12) by $\tilde{W}(z)$ yields

$$K(\xi) = \frac{2\pi^{3/2} q d}{\delta\rho} \sum_{j\alpha} \tau_{j\alpha}^2 \nu_{j\alpha}^4 G(\xi\sigma). \qquad (17)$$

By substituting (17) into (4) and introducing a new variable we obtain [2]

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\varepsilon(\lambda) = \frac{R^*}{2}\int_0^\sigma G(|\lambda-\mu|)\varepsilon(\mu)\,\mathrm{d}\mu + (1-R^*)\varepsilon^0 + \tau\psi(\lambda). \qquad (18)$$

in which

$$R^* = \frac{1/\tau^*}{1/\tau^* + 1/\tau^0}, \quad \lambda = \sigma\xi. \qquad (19)$$

It is noteworthy in this case, that equation (18) coincides formally with the known Biberman–Holstein equation [3, 4], describing the function of distribution of the excited two-level atoms at resonance scattering in a spectral line in the approximation of complete redistribution by frequencies. We introduce now the mean absorption coefficient, k_c, in the centres of the lines

$$k_c \equiv \frac{\sigma}{\delta} = \sqrt{2}\,\frac{\int_{\Delta\nu} k_\nu^2 \nu^3\,\mathrm{d}\nu}{\int_{\Delta\nu} k_\nu \nu^3\,\mathrm{d}\nu}. \qquad (20)$$

To derive formula (20), use should be made of equations (I. 24–27), (8), (16). Note that for a three-dimensional case we have the following equation

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\varepsilon(\mathbf{r}) = \frac{R^*}{4\pi}\int_V \mathrm{d}\mathbf{r}'\varepsilon(\mathbf{r}')\int_{-\infty}^{+\infty} \frac{k_c}{\sqrt{\pi}}\alpha^2(\omega) \times \frac{\exp(-\alpha(\omega)k_c|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^2}\,\mathrm{d}\omega + \tau\Phi(\mathbf{r}, t), \qquad (21)$$

in which

$$\Phi(\mathbf{r}, t) = \tau^{-1}(1-R^*)\varepsilon^0(\mathbf{r}, t) + \psi(\mathbf{r}, t). \qquad (22)$$

As is easy to verify, for the individual case of the volume V being a flat layer, equation (21) is transformed into (18).

We introduce now the following approximation

$$F(z) \equiv \pi^{-1/2}\int_{-\infty}^{+\infty} \alpha^2(\omega)\exp(-\alpha(\omega)z)\,\mathrm{d}\omega \simeq \frac{1}{\sqrt{2}}\exp\left(-\frac{z}{\sqrt{2}}\right). \qquad (23)$$

For justification of (23) we introduce the functions

$$\tilde{F}(z) = \pi^{-1/2}\int_{-\infty}^{+\infty} \tilde{\alpha}^2(\omega)\exp[-\tilde{\alpha}(\omega)z]\,\mathrm{d}\omega,$$

$$\tilde{\alpha}(\omega) = \begin{cases} b, & \omega \in [-a, a] \\ 0, & \omega \bar{\in} [-a, a] \end{cases} \qquad (24)$$

then, if the following conditions are imposed

$$F(0) = \tilde{F}(0), \quad \int_0^\infty F(z)\,\mathrm{d}z = \int_0^\infty \tilde{F}(z)\,\mathrm{d}z, \qquad (25)$$

it is easy to determine, that $a = \sqrt{\pi/2}$, $b = 1/\sqrt{2}$.

Assuming $F(z) \simeq \tilde{F}(z)$, we obtain (23). It was in a slightly different way that equation (23) was introduced in [5]. Thus, the approximation (23) is equivalent to the replacing of the line-shape function $\alpha(\omega)$ by $\tilde{\alpha}(\omega)$, which, according to (24), can be reasonably called a monochromatic one. The approximation (23) reduces equation (21) to

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\varepsilon(\mathbf{r}) = k\frac{R^*}{4\pi}\int_V \frac{\exp(-k|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^2}\varepsilon(\mathbf{r}')\,\mathrm{d}\mathbf{r}' + (1-R^*)\varepsilon^0(\mathbf{r}) + \tau\psi(\mathbf{r}, t). \qquad (26)$$

in which $k = k_c/\sqrt{2}$

$$k = \frac{\int_{\Delta\nu} k_\nu^2 J_\nu^0\,\mathrm{d}\nu}{\int_{\Delta\nu} k_\nu J_\nu^0\,\mathrm{d}\nu}. \qquad (27)$$

Hence, the frequencies average absorption coefficient k is a mean value of k_ν with the weight function $k_\nu J_\nu^0$. Note that equation (26) coincides formally with the master kinetic equation of the neutron transport theory.

Using (I. 24–27), equation (27) can be written as

$$k = \frac{(hc)^2 m^{1/2} B_e BN}{2\pi^{1/2}(k_B T)^{3/2}} \frac{\sum_{\alpha=0}^{1} \int_0^{\infty} \mathrm{d}j v_{j\alpha}^4 (j+\alpha)^2 \exp\left\{-2j(j+1)\frac{hcB_e}{k_B T}\right\}}{\sum_{\alpha=0}^{1} \int_0^{\infty} \mathrm{d}j v_{j\alpha}^4 (j+\alpha) \exp\left\{-j(j+1)\frac{hcB_e}{k_B T}\right\}}, \tag{28}$$

in which summation over j is substituted by integration. Approximate integration in (28) allows a sufficiently accurate formula to be obtained

$$k = 2^{-7/2}(hc)^{3/2}(mB_e)^{1/2}\frac{NB}{k_B T}. \tag{29}$$

It is obvious that k differs essentially from corresponding mean coefficients introduced according tò Plank and Rosseland.

Validity of the approach, based on the mean coefficient k (29), is limited by the condition

$$\exp\left(-\frac{h\nu_0}{k_B T_v}\right) \ll 1, \tag{30}$$

which is being satisfied with good accuracy in many of the problems of diatomic gas thermal radiation. Calculation of k (27) with allowance for anharmonicity leads to the appearance of an extra factor depending on vibrational temperature. When condition (30) is satisfied, the mean coefficient k does not depend on T_v. A more general statement is obvious: compliance with condition (30) is required for using the spectral absorption coefficient k_ν (I. 24) in equation (1), where no allowance is made for the molecules anharmonicity.

2. DIFFUSION APPROXIMATION

In lieu of the radiative transfer equation (I. 22), we use now the two-flux approximation [6], then, according to [7], a complete set of equations with allowance for (I. 40) can be written as

$$\frac{\partial}{\partial t}\varepsilon = \frac{\varepsilon^0 - \varepsilon}{\tau^0} - \rho^{-1}\nabla \int_{\Delta\nu} \mathbf{S}_\nu \,\mathrm{d}\nu, \tag{31}$$

$$\nabla \mathbf{S}_\nu = ck_\nu (U_\nu^0 - U_\nu), \tag{32}$$

$$\mathbf{S}_\nu = -\frac{c}{4k_\nu}\nabla U_\nu, \tag{33}$$

in which

$$U_\nu^0 = \frac{1}{c}\int_{(4\pi)} J_\nu^0 \,\mathrm{d}\Omega$$

$$= \frac{8\pi h\nu^3}{c^2}\left[\exp\left(\frac{h\nu_0}{k_B T_v}\right) - 1\right]^{-1}, \tag{34}$$

$$U_\nu^0 = \frac{8\pi\nu^3 m}{c^3 \nu_0}\varepsilon. \tag{35}$$

It is also necessary to specify the boundary condition, being a consequence of the two-flow approximation, as well as the initial condition

$$\mathbf{S}_\nu\bigg|_\partial = \mathbf{n}\frac{cU_\nu}{2}\bigg|_\partial, \quad \varepsilon(\mathbf{r}, t)\bigg|_{t=0} = E(\mathbf{r}). \tag{36}$$

Here $\mathbf{n}$ is the external normal of the boundary of a radiating volume.

Integrating equations (32), (33), (35) over the frequency within the limits of the effective width of the vibrational–rotational band and introducing the notations

$$\mathbf{S} = \int_{\Delta\nu} \mathbf{S}_\nu \,\mathrm{d}\nu, \quad U = \int_{\Delta\nu} U_\nu \,\mathrm{d}\nu, \quad a = \frac{8\pi m\nu_0^2 \Delta\nu}{c^3}, \tag{37}$$

one obtains from the set (31)–(35)

$$\frac{\partial}{\partial t}\varepsilon = \frac{\varepsilon^0 - \varepsilon}{\tau^0} - \frac{1}{\rho}\nabla\mathbf{S}, \tag{38}$$

$$\nabla\mathbf{S} = c\kappa(a\varepsilon - U), \tag{39}$$

$$\mathbf{S} = -\frac{c}{4\kappa}\nabla U, \tag{40}$$

$$\mathbf{S}\bigg|_\partial = \mathbf{n}\frac{cU}{2}\bigg|_\partial, \quad \varepsilon\bigg|_{t=0} = E(\mathbf{r}). \tag{41}$$

where κ is a certain unknown effective absorption coefficient.

Representing the integral density of the radiative flux $\mathbf{S}$ in terms of ε, we find with the help of (38)–(40)

$$\mathbf{S} = -\frac{c}{4\kappa}\nabla\left[a\varepsilon + \frac{\rho}{c\kappa}\left(\frac{\partial\varepsilon}{\partial t} - \frac{\varepsilon^0 - \varepsilon}{\tau^0}\right)\right]. \tag{42}$$

Substitution of (42) into the RHS of (38) yields the following equation for the density of vibrational energy

$$\left(\frac{\partial\varepsilon}{\partial t} + \frac{\varepsilon}{\tau^0}\right) = \alpha^2\nabla^2\left(\frac{\partial\varepsilon}{\partial t} + \frac{\varepsilon}{\tau}\right) + \frac{\varepsilon^0}{\tau^0}, \tag{43}$$

in which

$$\frac{1}{\tau} = \frac{1}{\tau^0} + \frac{1}{\tau^*}, \quad \tau^* = \frac{\rho}{ac\kappa}, \quad \alpha = \frac{1}{2\kappa}. \tag{44}$$

To obtain a boundary condition for equation (43), we express first the radiant energy density U in terms of ε

$$U = a\tau^*\left(\frac{\partial\varepsilon}{\partial t} + \frac{\varepsilon}{\tau} - \frac{\varepsilon^0}{\tau^0}\right). \tag{45}$$

Then, exclusion of the radiation flux density from (40) and (41) yields the boundary condition for U

$$(U + \alpha\mathbf{n}\nabla U)|_\partial = 0. \tag{46}$$

Substituting (45) into (46), we obtain the boundary condition for ε

$$\left[\left(\frac{\partial\varepsilon}{\partial t} + \frac{\varepsilon}{\tau}\right) + \alpha\mathbf{n}\nabla\left(\frac{\partial\varepsilon}{\partial t} + \frac{\varepsilon}{\tau}\right)\right]\bigg|_\partial = \frac{\varepsilon^0}{\tau^0}. \tag{47}$$

Besides, with the help of (41) and (46) we obtain the formula for the integral radiation flux from the surface of a radiating volume

$$\mathbf{S}\Big|_{\partial} = \alpha\rho\mathbf{n}\left(\frac{\partial\varepsilon}{\partial t}+\frac{\varepsilon}{\tau}-\frac{\varepsilon^0}{\tau^0}\right)\Big|_{\partial} \tag{48}$$

The disadvantage of the diffusion approximation, based on (43) and (47), lies in the uncertainty of κ and, consequently, of the diffusion coefficient α^2. However, it is possible to overcome this difficulty by using the above equation (26). Actually, in the flat layer case equation (26) assumes the form

$$\left(\tau\frac{\partial}{\partial t}+1\right)\varepsilon(x) = k\frac{R^*}{2}\int_0^\delta E_1(k|x-y|)\varepsilon(y)\,\mathrm{d}y + (1-R^*)\varepsilon^0+\tau\psi(x,t). \tag{49}$$

To obtain the diffusion equation and the corresponding boundary condition, it would suffice to use the conventional approximation of the exponential integral function $E_1(x) \simeq \sqrt{3}\exp(-\sqrt{3}x)$. Substituting this expression into (49) we find

$$\left(\tau\frac{\partial}{\partial t}+1\right)\varepsilon(x) = \frac{\sqrt{3}}{2}kR^*\int_0^\delta \exp(-\sqrt{3}k|x-y|)\varepsilon(y)\,\mathrm{d}y + (1-R^*)\varepsilon^0+\tau\psi(x,t). \tag{50}$$

Differentiation of (50) yields

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau^0}\right)\varepsilon(x) = \frac{1}{3k^2}\frac{\partial^2}{\partial x^2}\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\varepsilon(x) + \frac{\varepsilon^0}{\tau^0}+\left(1-\frac{1}{3k^2}\frac{\partial^2}{\partial x^2}\right)\psi, \tag{51}$$

$$\left[\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\varepsilon(x)\mp\frac{1}{\sqrt{3}k}\frac{\partial}{\partial x}\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\varepsilon(x)\right]\Bigg|_{x=0,\delta} = \frac{\varepsilon^0}{\tau^0}+\left(1\mp\frac{1}{\sqrt{3}k}\frac{\partial}{\partial x}\right)\psi\Bigg|_{x=0,\delta}. \tag{52}$$

Comparing (43), (47) with (51), (52), respectively, we determine $\alpha = 1/\sqrt{3}k$ and $\kappa = (\sqrt{3}/2)k$. From (51), (52) it follows that, to allow for the external source $\psi(\mathbf{r}, t)$ in the three-dimensional case, it is necessary to add the terms $[1-(\alpha\nabla)^2]\psi$ and $(1+\alpha\mathbf{n}\nabla)\psi|_\partial$ to the RHS of (43) and (47), respectively.

Analytical solutions of diffusion equation (43) with arbitrary initial distribution of $E(\mathbf{r})$, and with external source $\psi(\mathbf{r}, t)$ for geometries, such as a cylinder, a sphere and a flat layer, have been obtained in [7, 8].

To determine the accuracy of the diffusion approximation, an attempt has been made in co-operation with L. V. Katkovsky and L. V. Khodyko, to numerically solve equation (4), with allowance for 200 vibrational–rotational transitions being made. The results demonstrated a good agreement with the solution of the system (51), (52) at $R^* \lesssim 0.99$ for any optical layer depth.

In conclusion we note, that the frequences averaged coefficient k (27), (29), introduced for calculation of the source function (I. 45), is also connected with the integral radiation intensity. Really, using (I. 24–27) and the relation

$$\pi^{-1/2}\int_{-\infty}^{+\infty}\alpha(\omega)\exp(-\alpha(\omega)z)\,\mathrm{d}\omega \simeq \exp\left(-\frac{z}{\sqrt{2}}\right), \tag{53}$$

which is obtained by integrating equation (23), it is easy to determine, that

$$J \equiv \int_{\Delta\nu} J_\nu\,\mathrm{d}\nu = \frac{\rho}{4\pi\tau^*}\int_0^\delta \varepsilon(s)\exp(-ks)\,\mathrm{d}s. \tag{54}$$

Equation (54) is valid if condition (30) and also $k\delta \leqq 1$ are complied with, because, in contrast to (23), the approximation (53) is valid only at $z \leqq 1$.

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APPENDIX

Extension of equation (21) to the space non-homogeneous case, when $\mathbf{v}$, T, ρ are arbitrary functions of $\mathbf{r}$ and t, is given by

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla+\frac{1}{\tau}\right)\varepsilon(\mathbf{r}) = \int_V \tilde{G}(\mathbf{r},\mathbf{r}')\varepsilon(\mathbf{r}')\,\mathrm{d}\mathbf{r}'+\frac{\varepsilon^0(\mathbf{r})}{\tau^0}+\psi(\mathbf{r},t), \tag{55}$$

where

$$\tilde{G}(\mathbf{r},\mathbf{r}') = \sqrt{\frac{2}{\pi}}\frac{q}{\rho(\mathbf{r})}\int_0^\infty \nu^3 k_\nu(\mathbf{r})k_\nu(\mathbf{r}')\,\mathrm{d}\nu \times\int_{-\infty}^{+\infty}\alpha^2(\omega)\frac{\exp(-\alpha(\omega)\tau_c(\mathbf{r},\mathbf{r}'))}{|\mathbf{r}-\mathbf{r}'|^2}\,\mathrm{d}\omega. \tag{56}$$

$$\tau_c(\mathbf{r},\mathbf{r}') = (hc)^{3/2}\left(\frac{B_e}{m}\right)^{1/2}\frac{B}{8k_B}\int_0^{|\mathbf{r}-\mathbf{r}'|}\frac{\rho(s)}{T(s)}\,\mathrm{d}s. \tag{57}$$

Indeed, it is easy to demonstrate that, in the case when $\mathbf{v} = 0$, $T = \text{const}$, $\rho = \text{const}$, the system (55)–(57) is equivalent to (21). Besides, it should be noted, that from all the possible extensions of (21) to the space non-homogeneous case, equation (55) is most similar by its functional structure to (I. 46). In particular, the function $\rho(\mathbf{r})\tilde{G}(\mathbf{r},\mathbf{r}')$, as well as $\rho(\mathbf{r})G(\mathbf{r},\mathbf{r}')$, is symmetrical with respect to the variables $\mathbf{r}, \mathbf{r}'$.

Integrating in (56) over the frequency v yields

$$\tilde{G}(\mathbf{r},\mathbf{r}') = (hc)^{5/2}\left(\frac{B_e}{m}\right)^{1/2}\frac{\omega_e^3 B^2}{k_B} \times \frac{\rho(\mathbf{r}')(T(\mathbf{r})T(\mathbf{r}'))^{1/2}}{(T(\mathbf{r})+T(\mathbf{r}'))^2}\,\frac{F[\tau_c(\mathbf{r},\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|^2}. \quad (58)$$

In the monochromatic approximation (23) it is obvious that

$$\tilde{G}_0(\mathbf{r},\mathbf{r}') = (hc)^{5/2}\left(\frac{B_e}{m}\right)^{1/2}\frac{\omega_e^3 B^2}{k_B} \times \frac{\rho(\mathbf{r}')(T(\mathbf{r})T(\mathbf{r}'))^{1/2}}{(T(\mathbf{r})+T(\mathbf{r}'))^2}\,\frac{\exp\left(-\frac{1}{\sqrt{2}}\tau_c(\mathbf{r},\mathbf{r}')\right)}{\sqrt{2}|\mathbf{r}-\mathbf{r}'|^2}. \quad (59)$$

Moreover, for the non-homogeneous case, equation (55) with the kernel $\tilde{G}_0(\mathbf{r},\mathbf{r}')$ instead of $\bar{G}(\mathbf{r},\mathbf{r}')$ is equivalent to (26). These results can also be generalized for the Voigt line profile [8].

It should be noted that, to determine the spectral intensity of radiation, it is necessary first to determine T_v, with the help of (55), (I. 44), for example, which makes it possible to use the equation of radiative transfer (I. 11) for determination of $J(\mathbf{k})$. A more simple equation (I. 22) can not be used here because of the roughness of the superpositional approximation, though its use for determination of the source function is justifiable. This fact can be explained by $\varepsilon(\mathbf{r})$ being, in contrast with $J(\mathbf{k})$, much less sensitive to variation of k_ν, which has made it possible to introduce the mean coefficient for calculation of the source function.

RAYONNEMENT DANS LES GAZ DIATOMIQUES EN NON-EQUILIBRE DE VIBRATION—II

Résumé—La seconde partie traite des approximations de l'équation de distribution d'énergie de vibration pour un gaz diatomique. On obtient les équations de type Riberman-Holstein, dans lesquelles on décrit approximativement le déséquilibre. L'équation de diffusion est obtenue et sont formulées les conditions aux limites correspondantes. On calcule le coefficient de diffusion qui, relié au coefficient moyen d'absorption, diffère essentiellement des valeurs moyennes introduites selon Planck et Rosseland.

VIBRATIONS-UNGLEICHGEWICHTS-STRAHLUNG VON ZWEIATOMIGEN GASEN, TEIL II

Zusammenfassung—Der zweite Teil behandelt die Näherung der Grundgleichung für die Schwingungsenergieverteilung eines zweiatomigen Gases. Man erhält Gleichungen des Biberman-Holstein Typs, mit denen annähernd Nichtgleichgewichte beschrieben wurden. Die Diffusionsgleichung wird abgeleitet und die zugehörigen Randbedingungen formuliert. Der Diffusionskoeffizienten bezogen. Unterschiede ergeben sich im wesentlichen zu den von Planck und Rosseland eingeführten Mittelwerten.

КОЛЕБАТЕЛЬНО-НЕРАВНОВЕСНОЕ ИЗЛУЧЕНИЕ ДВУХАТОМНЫХ ГАЗОВ — 2

Аннотация — Во второй части рассмотрены аппроксимации основного уравнения для распределения колебательной энергии двухатомного газа. Получены уравнения типа Бибермана-Холстейна и Пайерлса, приближенно описывающие неравновесность. Выведено диффузионное уравнение, а также соответствующее граничное условие. Вычислен коэффициент диффузии, который связан со средним коэффициентом поглощения, существенно отличающемся от соответствующих средних вводимых по Планку и Росселанду.